1st Balkan Physics Olympiad – 2019 BPO
July 14-18, Thessaloniki, Greece

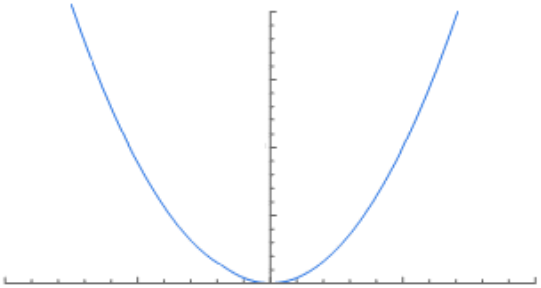
Problem 1 - Gravitational Billiard

A point particle is moving in a homogeneous gravitational field close to the Earth surface in a vertical plane limited from below by a parabola (as shown in the figure). Consider periodic motion in a gravitational billiard. A periodic motion repeats itself indefinitely when friction forces are absent. After reflection from the boundary the energy of the particle remains unchanged.

a) Draw the periodic trajectories that touch the boundary in one, two and three distinct points. **[3x3 points]**

b) Calculate the period for the first two cases. [One touching point - **6 points**]
[Two touching points - **10 points**]

The angle α between the tangent of the parabola $y = ax^2 + bx + c$ at an arbitrary point x_0 and the x-axis is given by $\tan \alpha = 2ax_0 + b$.



Problem 2

Two identical bodies A and B lie on a smooth horizontal surface. The two bodies are connected by a spring, with negligible mass, that obeys the Hooke's Law, with spring constant k and relaxed length L_0. A third body C, is suspended to the body B, by means of an ideal string (non-extensible and of negligible mass), passing over a smooth pulley P. The three bodies have the same mass m. Initially, the three-body system is at rest and the spring has its relaxed length. The system is accordingly released from rest.

a) Draw a free body diagram for each body after the system is released **[3 points]**

b) Calculate the minimum and maximum distance between the bodies A and B. Mind that during the three-body motion, body B doesn't hit on the pulley and body C doesn't reach the ground. **[10 points]**

c) Calculate the minimum value for the friction coefficient as well as the distance between the bodies A and B, in order for the system to remain at rest when it is released. **[6 points]**

d) Calculate the value of the friction coefficient as well as the distance between the bodies A and B in order for the system to move at constant speed. **[6 points]**