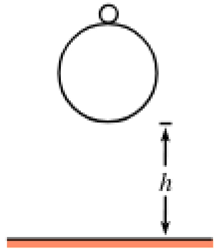
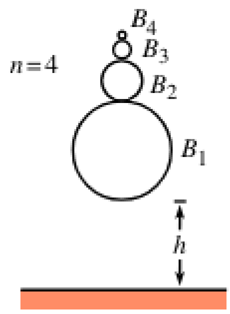
Problem 1 (25 points)



A) A tennis ball with the mass m_2 sits on top of a basketball with the mass m_1. The bottom of the basketball is a height h above the ground, and the bottom of the tennis ball is at a height $h + d$ above the ground (see the first figure). The balls are dropped to the ground. To what height does the tennis ball bounce?

Note: Work in the approximation where m_1 is much larger than m_2 and assume that the balls bounce elastically.

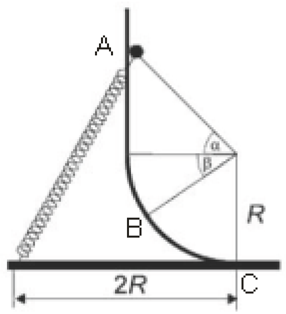


B) Now consider n balls, $B_1,\cdots, B_n$, having masses $m_1, m_2,\cdots, m_n$ (with $m_1 \gg m_2 \gg \cdots \gg m_n$), sitting in a vertical stack (see the second figure). The bottom of B_1 is at a height h above the ground, and the bottom of B_n is a height $h + l$ above the ground. The balls are dropped. In terms of n, to what height does the top ball bounce? If $h = 1.0$ m, what is the minimum number of balls needed for the top one to bounce to a height of at least 1.0 km? The same question, if the top ball is to reach the Earth's escape speed?

Note: Work in the approximation where m_1 is much larger than m_2, which is much larger than m_3, etc., and assume that the balls bounce elastically. Ignore wind resistance, etc., and assume that l is negligible.

Data: l is not very large, $R_{Earth} = 6.4 \cdot 10^6$ m, $g = 9.8\ \mathrm{m/s^2}$, .

Problem 2 (25 points)



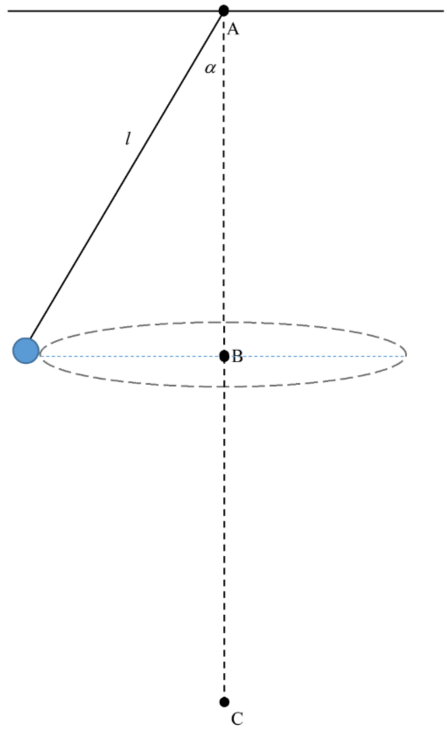
A quarter of a circle connects the vertical and the horizontal parts of a track. The radius of circular arc is $R = 20$ cm. A ball of unknown mass m slides on the track with no friction. The ball is connected (through a slit along the track) with a stretched spring, as is shown in the figure. The length of the unstretched spring is $L_0 = 0.20$ m, and the spring constant is $k = 100$ N/m.

The sliding ball starts with zero initial speed from a point that is $\alpha = 45°$ above the horizontal line when viewed from the centre of the arc, i.e. starts from point A, as is shown in the figure. The ball reaches the maximum speed at the angle $\beta = 34°$ below the horizontal line, at point B, as is shown in the figure ($g = 9.81\ \mathrm{m/s^2}$). Derive the mathematical expression and calculate the numerical value for:

a. **(3 points)** the length of the spring when the ball is at the starting position, at point A;
b. **(1 point)** the elongation of the spring when the ball is at the starting position at point A;
c. **(4 points)** the length of the spring when the ball is at the position of maximal speed, at point B;
d. **(1 point)** the elongation of the spring when the ball is at the of maximal speed, at point B;
e. **(2 points)** the magnitude of the spring force when the ball has maximal speed, at point B;
f. **(2 points)** the acceleration component of the ball, tangential to the track at point B;
g. **(5 points)** the mass of the ball;
h. **(4 points)** the maximum speed of the ball (at point B);
i. **(3 points)** the speed of the ball at point C.

Problem 3 (25 points)

A point-like ball of mass $m = 1.0\ \mathrm{g}$ is charged with an amount of positive charge $q = 1.0\ \mathrm{nC}$. It is suspended by a non-conducting thread of length $l = 0.50\ \mathrm{m}$. The mass of the thread is negligible. The ball rotates around a vertical axis in a circular path, the thread making an angle $\alpha = 30°$ with the vertical axis, as shown in figure.



An additional positive point-like charge $Q = 2.0\ \mathrm{nC}$ can be placed on the vertical axis in three different positions. In the figure those positions are marked with the letters A, B and C. The distance AB is equal with the distance BC.

The friction forces in the system can be neglected. The gravitational acceleration in the system is $g = 9.8\ \mathrm{m/s^2}$.

A. (9 points) When the additional charge is at point A:

- **a) (1 point)** Draw the free body diagram for the rotating ball.
- **b) (2 points)** Using the diagram write the equations of motion for the ball.
- **c) (2 points)** Derive the expression and calculate the numerical value for the tension force in the thread.
- **d) (2 points)** Assuming that the circular motion is uniform, derive the formula for the speed of the ball.
- **e) (2 points)** Derive the expression and calculate the period of rotation of the ball.

B. (8 points) Solve the tasks a, b, c, d and e when the charge Q is at point B.

C. (8 points) Solve the tasks a, b, c, d and e when the charge Q is at point C.

Problem 4 (25 points)

1. Description of the experiment

A long plastic spring, the so called "slinky", of mass $M = 72.3\ \mathrm{g}$, is composed of $N = 108$ identical circular turns. The slinky is positioned on a horizontal support, so that part of it is hanging freely, as depicted in Figure 1. After the spring oscillations damp completely, the length ℓ of the suspended part is measured as a function of the number n of hanging turns ($n < N$; see the figure). The experimental data for ℓ at different n are recorded in the Table at the end of the text.

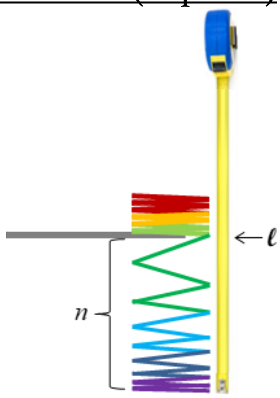


Figure 1

2. Tasks

A. (2 points) From a theoretical viewpoint, the dependence of ℓ on n is given by a quadratic function:

$$\ell = An + Bn^2 \qquad (1)$$

where A and B are constants characterizing the spring. Define a set of variables x and y, which linearize equation (1), i.e. transform it to a linear dependence.

B. (10 points) Determine constants A and B in equation (1) by means of a graphical analysis. Any auxiliary data, necessary for plotting the graph, must be recorded in the empty cells of Table 1. Use the graph paper to draw your graph. **Attach the last page of the text (containing the table) to your solution!**

C. (5 points) Equation (1) can be derived under the assumptions that each single turn of the slinky is equivalent to a spring of stiffness k_s (known also as "spring constant" or "coefficient of elasticity") and length ℓ_s in the undeformed state. Perform a theoretical analysis and express constants *A* and *B* in equation (1) in terms of k_s and ℓ_s. Neglect the deformation of a given single turn by its own weight.

D. (3 points) Calculate k_s and ℓ_s.

E. (5 points) Calculate the length *L* of whole slinky in the undeformed state, and its stiffness *K*.

Useful information:

1. The acceleration due to gravity is $g = 9.81\,\mathrm{m/s^2}$.
2. Depending on the methodology of your solution, the following relation may be of use:

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Attach this page to your solutions!

Table

n	ℓ/cm		
5	1.5		
10	3.9		
15	7.6		
20	12.9		
25	18.3		
30	25.0		
35	33.4		
40	42.8		
45	53.2		
50	64.6		