English

Problem №1: Hydrodynamic drag

Any object immersed in a fluid experiences a buoyancy (Archimedes) force $\vec{F}_\mathrm{b}$. If the object, however, moves in the fluid, additional drag (resistance) force $\vec{F}_\mathrm{d}$ acts on it, having the direction opposite to the direction of its velocity $\vec{v}$. For the conditions of the present problem, the magnitude of the drag force could be represented as:

$$F_\mathrm{d} = \frac{1}{2} C_\mathrm{d} \rho v^2 A$$

where ρ is the density of the fluid, A is the area of the widest cross section of the object, oriented perpendicular to the velocity (Fig. 1.1), and C_d is a dimensionless parameter, called *drag coefficient*. The drag coefficient is a characteristic of any particular shape of the object. The ultimate goal of this problem is to estimate the drag coefficient for a spherical object on the basis of experimental data regarding the motion of a metal ball in water.

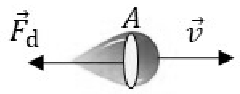


Figure 1.1

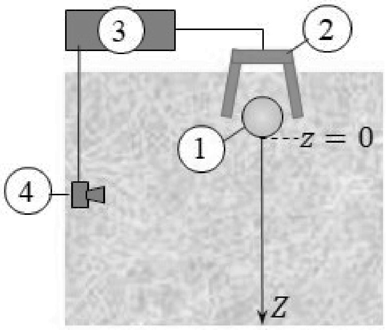


Figure 1.2

The experimental setup is shown schematically in Fig. 1.2. A metal ball (1) of unknown radius R and mass $m = 0{,}5$ kg is held under water in a swimming pool by means of electronically controlled clips (2). The clips open by an electric signal from a control block (3), which activates a digital camera (4), making a series of photos of the ball falling through water, with a repetition rate of $f = 5$ frames per second. There is an unknown time delay between dropping the ball and the camera activation. Based on the analysis of images, a series of ball positions z_i has been estimated and recorded in **Table 1** on the **Answer sheet**. The coordinate axis Z points vertically downwards with an origin $z = 0$ corresponding to the first recorded position of the ball.

You are given also a two sheets of **Graph paper** for data plots. **The used graph paper and the answer sheet with Table 1 must be supplied together with your solutions.**

A) (2 points) Make a sketch of all forces acting on the ball during its fall in water. Derive a theoretical formula, relating the instantaneous acceleration a of the ball to its instantaneous velocity v in terms of parameters and constants defined previously.

B) (8,5 points) Suppose that z_{i-1}, z_i, and z_{i+1} are the ball positions recorded at three equidistant moments of time, $t_{i-1} = t_i - \Delta t$, t_i and $t_{i+1} = t_i + \Delta t$, respectively. If the time interval Δt between the frames is small enough, the ball motion from t_{i-1} to t_{i+1} can be considered approximately as uniformly accelerated. Derive formulas, giving approximate values for the ball velocity v_i and acceleration a_i at the middle moment t_i. Append Table 1 with columns containing calculated approximate values of the velocity and the acceleration at the corresponding moments of time.

C) (3 points) Propose suitable auxiliary variables y and x, which linearize the acceleration–velocity relationship: $y = px + q$. Represent the available experimental data in these variables on a graph. By using a graphical approach estimate the parameters p and q of the suggested linear dependence.

D) (11,5 points) Estimate the density ρ_M of the metal ball and the drag coefficient C_d for an object of spherical form.

Some useful constants:

- Acceleration due to gravity, $g = 9{,}81\ \mathrm{m/s^2}$;
- Density of water, $\rho_\mathrm{w} = 1000\ \mathrm{kg/m^3}$.

English

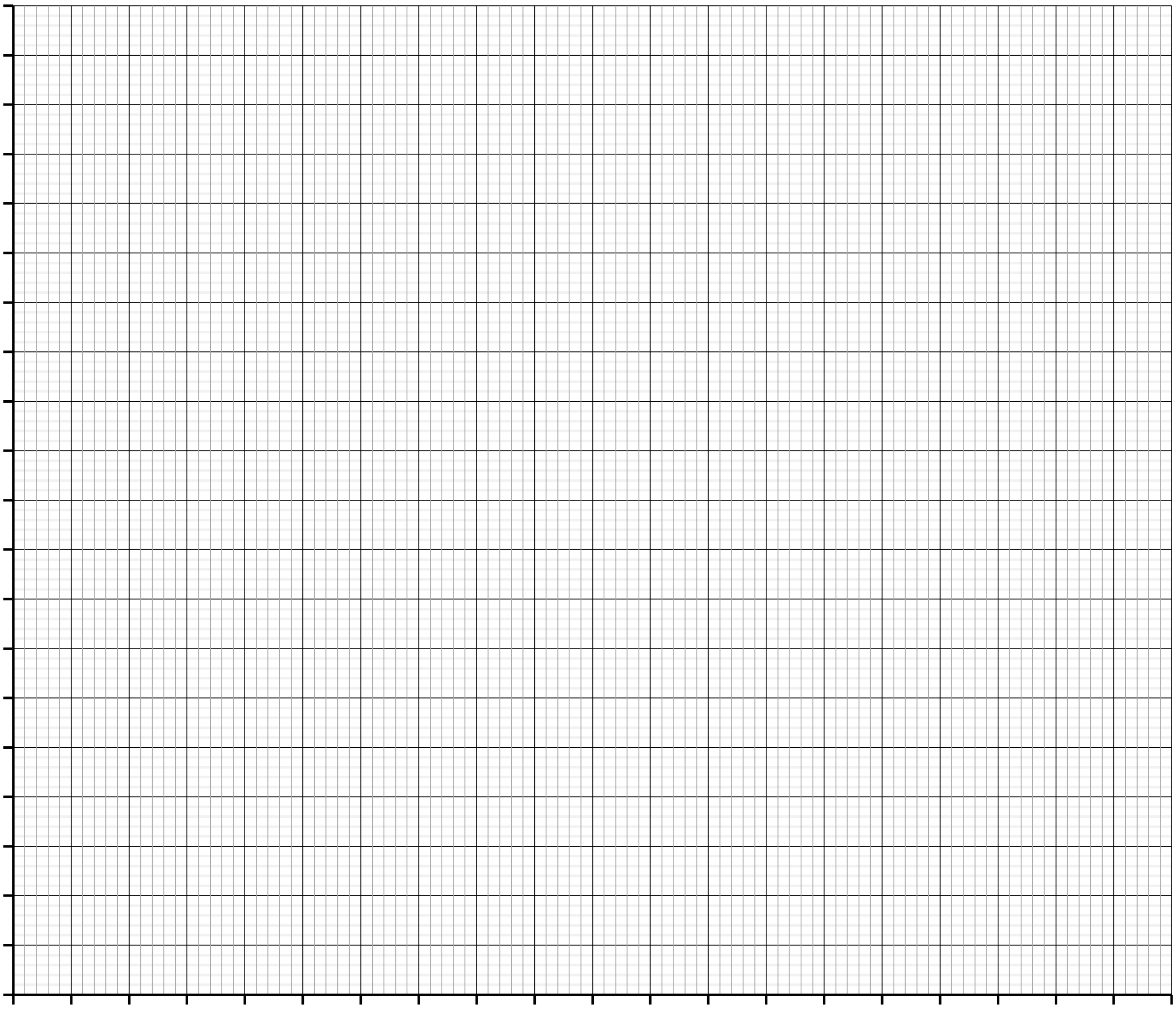
Answer sheet

Table 1. Fill-in calculation data in the empty cell, where appropriate. **Supply this sheet together with your solution.**

Frame #	z_i **(m)**			
1	0,00			
2	0,16			
3	0,43			
4	0,77			
5	1,13			
6	1,50			
7	1,86			
8	2,23			
9	2,59			
10	2,96			
11	3,33			
12	3,69			

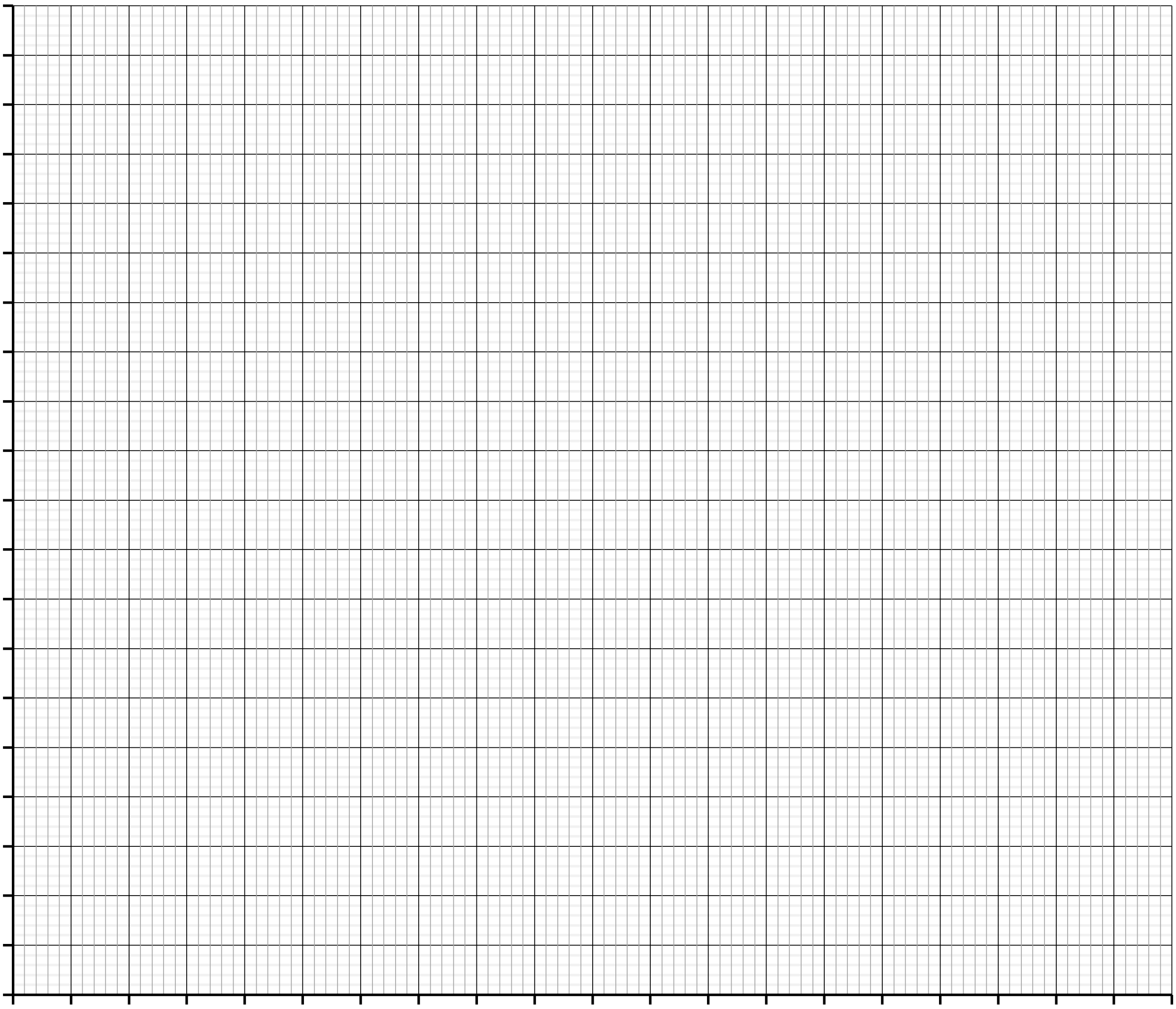
English

Supply this paper together with your solution



English

Supply this paper together with your solution



English

Problem №2: Trajectory

There is a planet without an atmosphere. The acceleration of free fall on its surface is $a = 10{,}0$ m/s^2. Two balls are thrown simultaneously from its surface from a point with Cartesian coordinates $(x, y) = (0{,}0)$ with one and the same speed v_0 (x is the horizontal axis, y is the vertical axis). The angles of throw for the first and second balls are respectively α and β. Two photos are made during their flight. The coordinates of the first ball on the first photo and on the second photo are respectively (x_1, y_1) and (x_2, y_2).

A) (6 points) Obtain a formula $y = f(x)$ for the trajectory of the first ball expressed in terms of the given quantities x_1, y_1, x_2, y_2.

B) (3 points) If $(x_1, y_1) = (5{,}0\text{ m}, 9{,}0\text{ m})$ and $(x_2, y_2) = (15{,}0\text{ m}, 21{,}0\text{ m})$, find the values of α (or its trigonometric function) and v_0.

C) (3 points) Obtain the formula for the horizontal distance L of the flight (expressed in terms of a, v_0 and α) of the first ball and calculate its value.

D) (2 points) It turned out that the flight distance L of the second ball is the same. Express β through α.

E) (3 points) Calculate the moments t_1 and t_2 (after the throw) when the first and second photo were taken.

F) (5 points) Calculate the coordinates (x'_1, y'_1) and (x'_2, y'_2) of the second ball on the first and second photo, respectively.

G) (3 points) Calculate how long it took the first ball to fall after the second (the time delay T).

English

Problem №3: Istanbul Bosphorus Cross-Continental Swim and current correction

The Istanbul Bosphorus Cross-Continental Swimming Race is an international open-water event in which swimmers cross from the Asian side of Istanbul to the European side. In this problem, the Bosphorus current is not neglected.

A simplified race course is modeled as a straight crossing of width

$$D = 1450\ \mathrm{m}$$

from point A on the Asian shore to point B on the European shore. The current flows parallel to the shores, from north to south, with constant speed $u = 0{,}65\ \mathrm{m/s}$. A swimmer swims with the constant speed $\mathrm{v} = 1{,}25\ \mathrm{m/s}$ relative to the water.

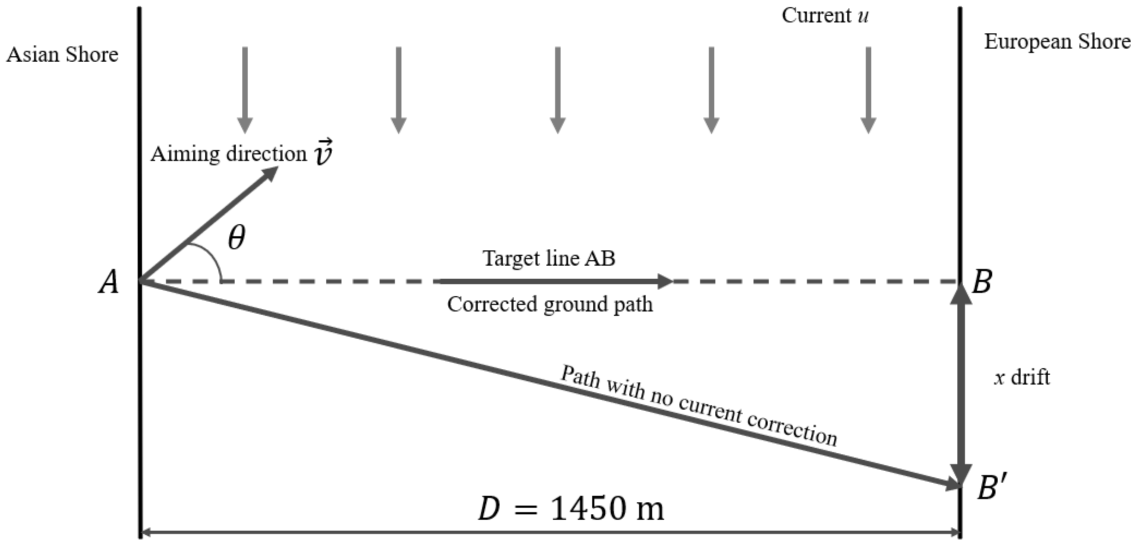


Figure 3.1 Current correction during the Istanbul Bosphorus crossing.

The swimmer wants to reach exactly point B, directly opposite to point A. Therefore, the swimmer must aim upstream at an angle θ with respect to the straight line AB.

A) (6 points) Find the angle θ at which the swimmer must aim relative to the line AB in order to arrive exactly at point B.

B) (6 points) Determine the time required for the swimmer to cross the Bosphorus and reach point B.

C) (6 points) A second swimmer ignores the current and swims directly toward point B. Determine the time needed to reach the European shore, as well as the downstream displacement from point B.

D) (7 points) During a training session, three swimmers cross the same route. Their measured crossing times and downstream displacements are given below.

Swimmer	Crossing time t (s)	Downstream displacement x (m)
S1	1320	0
S2	1160	270
S3	1450	-110

Here positive x means downstream from B, and negative x means upstream from B.

For each swimmer, determine the average ground-velocity components

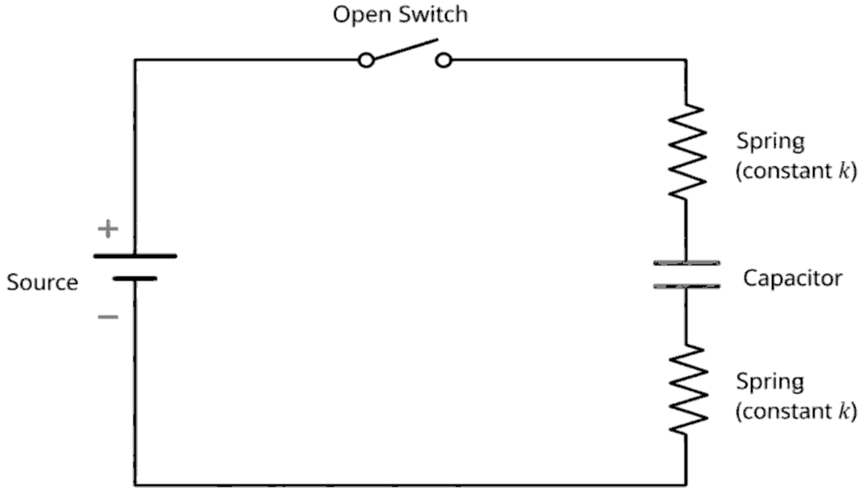
$$v'_x = \frac{x}{t}, \qquad v'_y = \frac{D}{t},$$

and decide which swimmer made the best current correction.

English

Problem №4: Spring circuit

The circuit below consists of two identical springs with elastic constant k, a capacitor of capacitance C, a voltage source ΔV, and a switch. The electric resistance of each spring is negligible as well as its magnetic properties.



When the switch is open, the capacitor is uncharged. It has capacitance C, and the distance between the plates is d. The space between the plates is vacuum with dielectric constant ε_0.

When the switch is closed, the plates move closer together, and their distance becomes $d/2$.

A) (2 points) Derive the expression for the charge of the capacitor in the final state.

B) (12 points) Derive the expression for the force that one of the plates exerts on the other.

C) (6 points) Find the spring constant (k) of the springs without using energy methods.

D) (5 points) Find the spring constant (k) of the springs using energy methods.